(i, j)-Step Competition Graphs Thus Far

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Computation Across Disciplines REU 2017 at Marquette University

June 13th, 2017
Overview

▶ Introduction

▶ What We’ve Learned

▶ Where We’re Going
Introduction

- Motivations: Explore \((i, j)\)-Step Competition Graphs and Their Applications to Ecology or Expand Their Theory
  - Background Readings
- Importance: Numerical Methods are (Sometimes) Easier and Not All Methods are Made Equal
What We’ve Learned

- LaTeX (this presentation)
- Competition graph
- Competition graph number
- (1, 2)-step competition graph
- (i, m)-step competition graph
- (1, 2)-step competition number
- In(out)-neighborhood of a vertex
- In(out)degree of a vertex
Competition Graph

- Competition graph: the *competition graph of* $D$, $C(D)$ is the graph with vertex set $V(D)$ such that $\{u, v\}$ is an edge if vertices $u$ and $v$ have a common prey in $D$. See the example below.

![Competition Graph Diagram]
Competition Number of a Graph

Competition number: the *competition number* \( k(G) \) of a graph \( G \) is the smallest nonnegative integer \( k \) so that \( G \), together with \( k \) isolated vertices, is the competition graph of some acyclic digraph.

Given:

\[
\begin{align*}
\text{1} & \quad \text{2} \\
\text{3} & \quad \text{4}
\end{align*}
\]

\[
\begin{align*}
\Rightarrow \quad & \v1 \\
\v4 & \quad \v2 \\
\v3 & \quad \v4
\end{align*}
\]

\[
\begin{align*}
\Rightarrow \quad & \v1 \\
\v4 & \quad \v2 \\
\v3 & \quad \v4
\end{align*}
\]

\[
\begin{align*}
\Rightarrow \quad & \v1 \\
\v4 & \quad \v2 \\
\v3 & \quad \v4
\end{align*}
\]
(1, 2)-step Competition Graph

(1,2)-step competition graphs or $C_{1,2}(D)$ is a graph with a vertex set $V(D)$ in which distinct vertices $x$ and $y$ will make an edge $\{x,y\}$ only when some vertex $(z) \in V(D)$, $d_{D-y}(x,y) \leq 2$ and $d_D(y,z) = 1$ or $d_D - x(y,z) \leq 2$ and $d_D(x,z) = 1$. 
(1, 2)-step Competition Graph

- Shown below is an example of a complete graph.
(1, 2)-step Competition Graph

To begin, two vertices can be looked at on the graph: (4, 2) along with a z vertex, 1. It can be observed that the distance between the y and z vertex is $\leq 2$ when looking at the graph minus x. It is also true that distance between x and z is one.
After going through every edge, this is the resulting (1, 2)-step competition graph.
The (1,2)-step competition graph can be generalized as a 
(i, m)-step competition graph: if for some $z \in V(T) - \{x, y\}$, 
$d_{T-y}(x, z) \leq i$ and $d_{T-x}(y, z) \leq m$ or 
$d_{T-x}(y, z) \leq i$ and 
$d_{T-y}(x, z) \leq m$.

This follows the same process as the (1,2)-step competition 
graph, but looks for values that are equal whatever i,m 
represent.
The $(1,2)$-step competition number of a graph $G$ is the minimum number $k$ such that $G$ with $k$ isolated vertices is the $(1,2)$-step competition graph of an acyclic digraph.

The $(1,2)$-step competition number is the $(1,2)$-step competition graph’s version of a competition number of a graph $G$. That is, we determine it for a specific graph by adding isolated vertices to the vertex set of $G$. Then, we redefine the arc set of $G$ to create a digraph such that our original $G$ is the $(1,2)$-step competition graph of $G$. 
$(i, m)$-step Competition Number

- Similar to the $(1,2)$-step competition number, the $(i, m)$-step competition number is the $(i, m)$-step competition graph’s version of a competition number of a graph $G$. It is a determination of what $i, m$ are and then we begin adding isolated vertices to the vertex set of $G$. The arc set is then redefined to create a digraph in which such that our original $G$ is the $(i, m)$-step competition graph of $G$. 
Out-Neighborhood and In-Neighborhood of a Vertex

- Out-Neighborhood of vertex $x$: the set of all vertices to which $x$ has an arc in $D$, denoted $N_D^+(x)$
- In-Neighborhood of a vertex $x$: the set of all vertices from which $x$ has an arc in $D$, denoted $N_D^-(x)$

\[ N_D^+(1) = \{2, 3\} \]
\[ N_D^+(2) = \{4\} \]
\[ N_D^+(3) = \{2\} \]
\[ N_D^+(4) = \{1, 3\} \]

\[ N_D^-(1) = \{4\} \]
\[ N_D^-(2) = \{1, 3\} \]
\[ N_D^-(3) = \{1, 4\} \]
\[ N_D^-(4) = \{2\} \]
# Outdegree and Indegree of a Vertex

- **Outdegree of a vertex** $x$: the cardinality of the out-neighborhood of $x$, denoted $\deg_D^+(x)$
- **Indegree of a vertex** $x$: the cardinality of the in-neighborhood of $x$, denoted $\deg_D^-(x)$

## Out-Neighborhoods: In-Neighborhoods:

<table>
<thead>
<tr>
<th>$N_D^+(1)$</th>
<th>$N_D^-(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${2, 3}$</td>
<td>${4}$</td>
</tr>
<tr>
<td>$N_D^+(2)$</td>
<td>$N_D^-(2)$</td>
</tr>
<tr>
<td>${4}$</td>
<td>${1, 3}$</td>
</tr>
<tr>
<td>$N_D^+(3)$</td>
<td>$N_D^-(3)$</td>
</tr>
<tr>
<td>${2}$</td>
<td>${1, 4}$</td>
</tr>
<tr>
<td>$N_D^+(4)$</td>
<td>$N_D^-(4)$</td>
</tr>
<tr>
<td>${1, 3}$</td>
<td>${2}$</td>
</tr>
</tbody>
</table>

## Outdegrees: Indegrees:

<table>
<thead>
<tr>
<th>$\deg_D^+(1)$</th>
<th>$\deg_D^-(1)$</th>
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<tbody>
<tr>
<td>$</td>
<td>N_D^+(1)</td>
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<tr>
<td>$\deg_D^+(2)$</td>
<td>$\deg_D^-(2)$</td>
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<tr>
<td>$</td>
<td>N_D^+(4)</td>
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</tbody>
</table>
Where We’re Going

- Generalization to hypergraphs

- Expanding on section 4 from ”The (1,2)-step competition number of a graph” by Factor et al.

- Experiment with colorings of graphs and their (”1,2”) -step competition graphs
Acknowledgements

- National Science Foundation
- Marquette University MSCS
- Dr. Kim A.S. Factor