(1,2)-Step Competition Graph Examples

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1 Introduction

This document is a web of examples constructed from the definitions found in The (1,2)-step competition number of a graph by Factor et al.

2 Definitions

Note: before continuing with the definitions, we let $D$ represent an arbitrary digraph, $V(D)$ be its vertex set, and $A(D)$ be its arc set. Furthermore, we let $G$ represent an arbitrary graph, $V(G)$ be its vertex set, and $E(G)$ be its edge set. We let $K_n$ denote a complete graph (digraph) of $n$ vertices; we will specify whether we are using a graph or digraph. These objects are what we will work with unless otherwise specified.

Definition 2.1. Competition graph: the competition graph of $D$, $C(D)$ is the graph with vertex set $V(D)$ such that \{u, v\} is an edge if vertices u and v have a common prey in $D$.

Definition 2.2. Competition number: the competition number $k(G)$ of a graph $G$ is the smallest nonnegative integer $k$ so that $G$, together with $k$ isolated vertices, is the competition graph of some acyclic digraph.

Definition 2.3. (1,2)-step competition graphs or $C_{1,2}(D)$ a graph with a vertex set $V(D)$ in which distinct vertices $x$ and $y$ will make an edge \{x,y\} only when some vertex $(z) \in V(D)$, $d_{D-y}(x,y) \leq 2$ and $d_{D}(y,z) = 1$ or $d_{D-x}(y,z) \leq 2$ and $d_{D}(x,z) = 1$. 

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Definition 2.4. The (1,2)-step competition graph can be generalized as a 
\((i,m)\)-step competition graph: if for some \(z \in V(T) - \{x,y\}\), 
\(d_{T-y}(x,z) \leq i\) and \(d_{T-x}(y,z) \leq m\) or \(d_{T-x}(y,z) \leq i\) and 
\(d_{T-y}(x,z) \leq m\).

Definition 2.5. Out-Neighborhood of vertex \(x\): the set of all vertices to
which \(x\) has an arc in \(D\), denoted \(N_D^+(x)\)

Definition 2.6. In-Neighborhood of a vertex \(x\): the set of all vertices from
which \(x\) has an arc in \(D\), denoted \(N_D^-(x)\)

Definition 2.7. Outdegree of a vertex \(x\): the cardinality of the
out-neighborhood of \(x\), denoted \(\text{deg}_D^+(x)\)

Definition 2.8. Indegree of a vertex \(x\): the cardinality of the
in-neighborhood of \(x\), denoted \(\text{deg}_D^-(x)\)

Definition 2.9. \((i,m)\)-step competition number: the \((i,m)\)-step competition
number of a graph \(G\) is the minimum number \(k\) such that \(G\) with \(k\) isolated
vertices is the \((i,m)\)-step competition graph of an acyclic digraph.

3 Lemmas

Lemma 3.1. For all positive integers \(n \geq 5, k_{(1,2)}(C_n) > 1\)

4 Propositions

Proposition 4.1. Let \(i,m\) be positive integers. For any graph
\(4G\), there exists an nonnegative integer \(k\) such that \(4G\) with \(k\) isolated
vertices in the \((i,m)\)-step competition graph of an acyclic digraph.

Proposition 4.2. Let \(i,m\) be positive integers. If a graph \(G\) has no isolated
vertices, then \(k_{(i,m)}(G) \geq 1\).

5 Theorems

Theorem 5.1. Let \(i,m\) be positive integers. For \(n \geq 2\), we have \(k_{(i,m)}(K_n) = 1\).

Theorem 5.2. For all positive integers \(n > 1, k_{(1,2)}(P_n)\).
**Theorem 5.3.** Let $n \geq 3$ be a positive integer. Then $k_{(1,2)}(C_n) = 1$ when $n \geq 3, 4$. For $n \geq 5, k_{(1,2)}(C_n) = 2$.

**Theorem 5.4.** Let $n$ be a positive integer. then $k_{(1,2)}(K_{1,n}) = 1$. 